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Analysis of a Class of Earth–Mars Cycler Trajectories

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Sun-orbiting spacecraft trajectories that repeatedly encounter Earth and Mars may play a central role in a future Earth–Mars transportation system. Such orbits are known as Earth–Mars cycler trajectories (cyclers). By using gravity-assist maneuvers at Earth or Mars, many cyclers can avoid using large amounts of propellant. The known cyclers were found using heuristics or numerical searches. We describe a new, systematic method for constructing and evaluating cyclers. Our method reveals that previously known cyclers, such as the Aldrin cycler and the Versatile International Station for Interplanetary Transport cyclers, belong to a larger family of cyclers. Our cycler construction method also reveals some previously unknown cyclers. For example, we identify a new cycler that repeats every two synodic periods and has a low V_∞ at Earth and Mars (5.65 and 3.05 km/s, respectively).

Nomenclature

a = semimajor axis, astronomical units (AU)
e = eccentricity
n = number of Earth–Mars synodic periods before repeating
$\mathcal{P}$ = orbit period, years
p = parameter of the cycler orbit, AU
$\boldsymbol{R}$ = position vector, AU
R = distance from sun to spacecraft, AU
R_a = aphelion radius, AU
R_p = perihelion radius, AU
r = number of complete revolutions before repeating
S = Earth–Mars synodic period, years
T = time to repeat the cycler trajectory, years
V_∞ = hyperbolic excess speed, km/s
$\Delta\Psi$ = angle between initial Earth position and Earth's position after n synodic periods, rad
ϕ = angle from Earth to Mars, rad
ω = argument of periapsis, rad

Introduction

IN the late 1950s and early 1960s, the utility of gravity-assist maneuvers was finally understood, and missions using multiple gravity-assist flybys[1–6] were shown to be possible. In the late 1960s, Hollister[7,8] and Hollister and Menning[9,10] discovered ballistic gravity-assist trajectories that repeatedly encounter Venus and Earth. Trajectories that repeatedly encounter the same planets on a regular schedule without stopping are now known as cycler trajectories, or cyclers.

Rall[11] and Rall and Hollister[12] appear to be the first to demonstrate that cycler trajectories exist between Earth and Mars. Their method of finding Earth–Mars cyclers was essentially heuristic, and so they wrote "Because of the cut-and-try nature of the method, one cannot be certain that all periodic [cycler] orbits have been found—even among the types of periodic orbits considered."[12] The cyclers they did find repeat every four synodic periods or more.

In 1985, Aldrin suggested that an Earth–Mars cycler may exist which repeats every synodic period.[13] The existence of this "Aldrin cycler" was subsequently confirmed by Byrnes et al.[14] Also in 1985, Niehoff first proposed the Versatile International Station for Interplanetary Transport (VISIT) 1 and VISIT 2 Earth–Mars cyclers.[15–17] These cyclers were investigated further and compared to the Aldrin cycler by Friedlander et al.[18] A natural question that arises is whether there are any other Earth–Mars cyclers. In this paper, we describe a new method of constructing such trajectories.

Assuming conic orbits, several researchers have investigated the families of trajectories that leave Earth (or any other orbiting body) and return at a later date.[19–26] Such trajectories are known as consecutive collision orbits. Howell and Marsh provide an excellent historical overview in Ref. 26. If only the Earth is used for gravity-assist maneuvers, then we can construct Earth–Mars cyclers by patching consecutive collision orbits together at the Earth encounters so that the entire trajectory repeats after an integer number of Earth–Mars synodic periods. We elaborate on this method of constructing Earth–Mars cyclers.

Interestingly, Poincaré knew about periodic solutions of this sort, that is, consecutive collision orbits patched together at planetary encounters.[27] Such orbits are known as Poincaré's second species periodic orbits and have been studied quite extensively.[28–33] However, as far as we know, Poincaré's second species periodic orbits have not previously been considered as potential Earth–Mars cycler trajectories.

Methodology

To construct Earth–Mars cycler trajectories, we begin by making a number of simplifying assumptions: 1) The Earth-Mars synodic period S is $2\frac{1}{7}$ years. 2) Earth's orbit, Mars's orbit, and the cycler trajectory lie in the ecliptic plane. 3) Earth and Mars have circular orbits. 4) The cycler trajectory is conic and prograde (direct). 5) Only the Earth has sufficient mass to provide gravity-assist maneuvers. 6) Gravity-assist maneuvers occur instantaneously. (For each cycler trajectory constructed in our simplified solar system model, we hope to find a corresponding cycler trajectory in a more accurate model.)

We note that assumption 1 is equivalent to assuming that the orbital period of Mars is $1\frac{7}{8}$ years (whereas a more accurate value is 1.881 years). Assumptions 2 and 3 allow us to set up a planar coordinate system with the sun at the origin and the Earth on the positive x axis on the launch date. (After we find a cycler trajectory,

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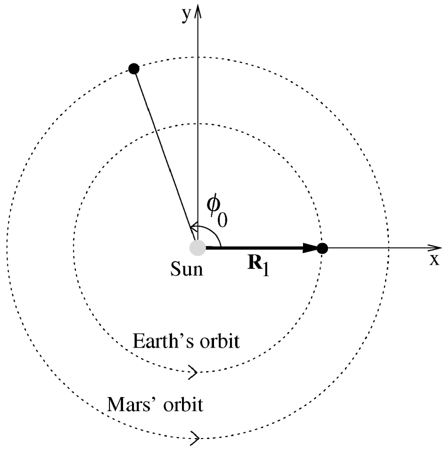


Fig. 1 Example initial configuration.

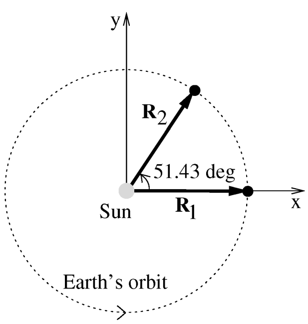


Fig. 2 Departure/arrival geometry when $n = 1, 8, 15, \ldots$.

we choose the launch date so that the spacecraft encounters Mars.) An initial Earth–Mars configuration is illustrated in Fig. 1.

Now we must determine what conditions must be met if the spacecraft orbit is to be a cycler trajectory. At the initial time, $t_0 = 0$, the angle ϕ_0 from Earth to Mars (as shown in Fig. 1) is chosen so that the spacecraft will encounter Mars after leaving Earth. Following the Mars encounter, the spacecraft may encounter Earth again. If an Earth encounter happens when the Earth–Mars angle $\phi = \phi_0$ again, then the spacecraft could return to Mars using the same (shape) Earth–Mars transfer orbit that it used initially. Moreover, the trajectory could be repeated indefinitely, and hence, it is a cycler trajectory.

Let T be the time to repeat a cycler trajectory. Then the preceding discussion implies that $\phi(T) = \phi(0) = \phi_0$. Because $\phi(t)$ (the angle from Earth to Mars) repeats once per synodic period, T must be an integer number of synodic periods:

$$T = nS = n \cdot \left(2\tfrac{1}{7}\right) \tag{1}$$

where $n = 1, 2, 3, \ldots$. Because the angular velocity of the Earth is 2π rad per year, we also know that $\boldsymbol{R}_{\mathrm{Earth}}(T) = [\cos(2\pi T), \sin(2\pi T)]$. Therefore, the conditions for the spacecraft orbit $\boldsymbol{R}(t)$ to be a cycler trajectory are

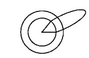
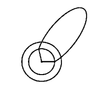
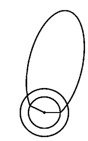
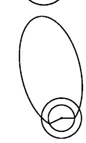
$$\boldsymbol{R}(0) = [1, 0] \tag{2}$$

$$\boldsymbol{R}(nS) = [\cos(2\pi nS), \sin(2\pi nS)] \tag{3}$$

where $n = 1, 2, 3, \ldots$. A problem of this form is known as a Lambert problem. Given n, we want to find a solution $\boldsymbol{R}(t)$ to the two-body problem that connects $\boldsymbol{R}_1 = [1, 0]$ to $\boldsymbol{R}_2 = [\cos(2\pi nS), \sin(2\pi nS)]$ in a time of flight $T = nS$.

For example, let us consider the case $n = 1$, which means we are looking for cycler trajectories that repeat every $T = nS = 2\frac{1}{7}$ years. In $2\frac{1}{7}$ years, the Earth orbits the sun $2\frac{1}{7}$ times, so that when the spacecraft returns to Earth after $2\frac{1}{7}$ years, the Earth will be $\frac{1}{7}$ of a revolution (51.43 deg) ahead of where it was when the spacecraft left (also true when $n = 8, 15, \ldots$). The geometry of this Lambert problem is illustrated in Fig. 2.

Table 1 Unique-period solutions (nU0)

n	Solution, $r = 0$
1	
2	
3	
4	

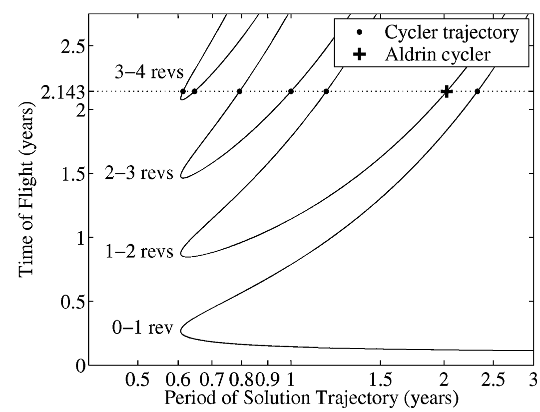


Fig. 3 Seven cycler trajectories that repeat every synodic period, $n = 1$.

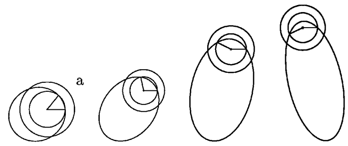
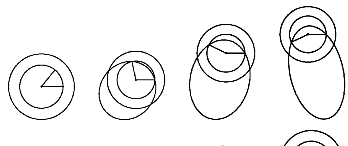
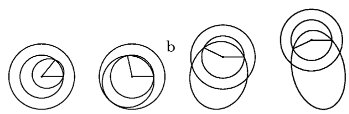
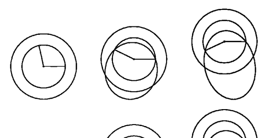
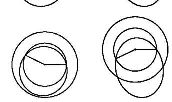
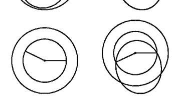
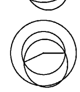
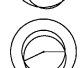
The $n = 1$ case has multiple solutions, that is, there are many different trajectories that connect $\boldsymbol{R}_1$ to $\boldsymbol{R}_2$ in $2\frac{1}{7}$ years. These solutions are illustrated in Fig. 3, which shows the orbital periods of solutions with various times of flight. The solutions to the $n = 1$ case correspond to those solutions with a time of flight of $2\frac{1}{7}$ years (2.143 years). In Fig. 3, we see that there are seven solutions, corresponding to the seven points on the solution curves with a time of flight of 2.143 years. In fact, one of the solutions is the Aldrin cycler, which has an orbital period of 2.02 years.

Categorizing Cycler Trajectories

As seen in the preceding example, there can be multiple solutions for a given choice of n (the time-to-repeat in synodic periods). For each n, there is one solution that makes less than one revolution. There are two solutions that make between one and two revolutions, two solutions that make between two and three revolutions, and so on. Once the number of revolutions is large enough, there are no solutions because the time of flight is not long enough to accommodate all of the revolutions. When there are two solutions for a given number of revolutions, they are referred to as the short-period and long-period solutions. Every solution can be uniquely identified by specifying 1) n, the time-to-repeat in synodic periods ($n = 1, 2, 3, \ldots$), 2) whether the solution is long period, short period, or unique period, that is, when $r = 0$, and 3) r, the number of revolutions, rounded down to the nearest integer [$r = 0, 1, 2, \ldots, r_{\max}(n)$].

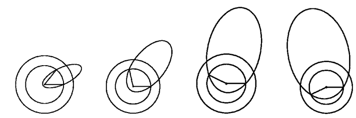
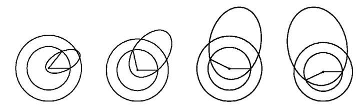
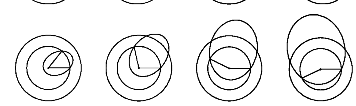
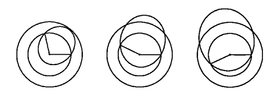
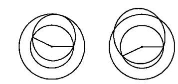
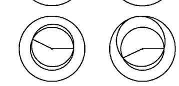
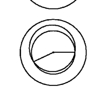
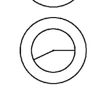
We denote a solution by a three-element expression of the form $n\mathrm{P}r$, where P is either L, S, or U depending on whether the solution is long, short, or unique period, respectively. For example, the seven solutions in the $n = 1$ case are 1U0, 1L1 (Aldrin cycler), 1S1, 1L2, 1S2, 1L3, and 1S3. Tables 1–3 show the form of the unique-, long-, and short-period

Table 2 Long-period solution (*n*L*r*)

Number of full revolutions, *r*	$n=1$	$n=2$	$n=3$	$n=4$
1	 (a)			
2				
3		(b)		
4	NS[c]			
5	NS	NS		
6	NS	NS		
7	NS	NS	NS	
8	NS	NS	NS	
9	NS	NS	NS	NS

[a]Aldrin cycler (see Ref. 14).
[b]Case 1 cycler analyzed by Byrnes et al.[34]
[c]No solution.

Table 3 Short-period solution (*n*S*r*)

Number of full revolutions, *r*	$n=1$	$n=2$	$n=3$	$n=4$
1				
2				
3				
4	NS[a] NS			
5	NS	NS		
6	NS	NS		
7	NS	NS	NS	
8	NS	NS	NS	
9	NS	NS	NS	NS

[a]No solution.

cyclers for $n=1$–4. The initial and final position vectors $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are indicated by line segments. Note that cyclers 1L2, 2L4, 3L6, and 4S8 are all equivalent to Earth's orbit.

When n is a multiple of seven, then $\boldsymbol{R}_1=\boldsymbol{R}_2$ (a resonant transfer), so that the Lambert problem becomes degenerate. In these cases, the nPr notation must be extended to accommodate the larger variety of solutions. We discuss this extension later.

Evaluating Solutions

Not all cycler trajectories are practical for applications. In this section we describe some criteria for evaluating their usefulness.

Number of Cycler Vehicles Required

When $n=1$ and the cycler trajectory crosses Mars's orbit, it crosses Mars's orbit at two points. By launching the cycler spacecraft at the correct time, it will encounter Mars at the first Mars-orbit crossing, which minimizes the time of flight from Earth to Mars. A cycler trajectory used in this way is called an outbound cycler because it is used to travel from Earth out to Mars. Similarly, the cycler spacecraft can be launched at a different time so that it encounters Mars at the last Mars-orbit crossing before returning to Earth (to minimize time of flight from Mars to Earth). When the cycler trajectory is used in this way it is called an inbound cycler. We note that the difference between an inbound cycler and an outbound cycler is the launch date, not the shape of the cycler trajectory.

For an outbound cycler, the short Earth–Mars leg (usually) occurs only once every $n\cdot(2\frac{1}{7})$ years (which is the repeat interval, T). For example, if a cycler repeats every two synodic periods, then the cycler vehicle will make the short Earth–Mars trip only once every two synodic periods. One synodic period after the cycler vehicle leaves Earth, the short Earth–Mars transfer becomes available again (at Earth), but the cycler vehicle is not there. By launching a second cycler vehicle at that time, that short Earth–Mars transfer opportunity can also be used. Therefore, two cycler vehicles are required to guarantee a short Earth–Mars trip every synodic period (for a two-synodic-period cycler). In general, when the cycler repeat time is n synodic periods, n cycler vehicles are required to take full advantage of all of the short Earth–Mars legs. Similarly, an additional n cycler vehicles are needed to take full advantage of all of the short Mars–Earth legs. Thus, a full fleet of cycler vehicles nominally has $2n$ vehicles. The value $2n$ is actually an upper bound because sometimes there is more than one short-duration Earth–Mars (or Mars–Earth) leg per repeat interval T. For example, the Rall[11] and Rall–Hollister cyclers[12] and the VISIT cyclers[15–17] have this property.

Aphelion Radius

For a cycler trajectory to be used for transportation between Earth and Mars, it should cross the orbit of Mars, that is, the aphelion radius should be greater than the orbital radius of Mars. A quick glance at Tables 2 and 3 reveals that some cycler trajectories do not pass this test. (For example, in Table 2, the cycler with $n=3$ and $r=5$ has an aphelion below Mars's orbital radius.) The eccentricity of Mars causes its heliocentric distance to vary between 1.38 and 1.67 astronomical units (AU), so that some of the cyclers with an aphelion radius not too far below the orbital radius of Mars still occasionally encounter Mars (if the encounter occurs when Mars is near perihelion).

In the cases where the spacecraft does not naturally encounter Mars, ΔV maneuvers could be used to force an encounter. Because of the cycler timing constraint, determining the optimal maneuvers is difficult and requires numerical optimization. The problem of optimizing the maneuvers for the low-aphelion cases is beyond the scope of the present analysis.

V_∞ at Earth and at Mars

Because taxi spacecraft must rendezvous with the cycler spacecraft as it passes Earth and Mars, we want the Earth V_∞ and the Mars V_∞ to be as small as possible, a requirement that often rules out trajectories with a small number of revolutions r per repeat interval. The orbit that achieves the lowest possible sum of V_∞ at Earth and V_∞ at Mars is the Hohmann transfer orbit (2.95 km/s at Earth and 2.65 km/s at Mars). Unfortunately, the Hohmann transfer orbit is not a cycler trajectory.

Required vs Maximum Possible Turn Angle

For the spacecraft to return to Mars on the same-shape orbit it used originally, the orbit's line of apsides must be rotated by $\Delta\Psi$ degrees, where $\Delta\Psi$ is the angle between the initial and final Earth positions ($\boldsymbol{R}_1$ and $\boldsymbol{R}_2$):

$$\Delta\Psi = (n/7)\cdot 360 \text{ deg } (\text{mod } 360 \text{ deg}) \tag{4}$$

We note that when n is a multiple of seven, the line of apsides does not need to be rotated (because $\Delta\Psi = 0$ deg). Therefore, all cycler trajectories with n a multiple of seven are ballistic cyclers, that is, cyclers that do not require any deterministic ΔV maneuvers. The VISIT 1 and VISIT 2 cyclers are examples of $n = 7$ solutions.

If the line of apsides must be rotated, an Earth gravity-assist may be able to accomplish the rotation without propellant. The required flyby radius must be sufficiently greater than the Earth's radius. We assume that Earth flybys are constrained to altitudes greater than or equal to 200 km. If the required altitude is less than 200 km, then a ΔV maneuver is needed.

Rotating the line of apsides is equivalent to rotating the V_∞ vector at Earth. If the required V_∞ turn angle is less than the V_∞ turn angle obtainable with a 200 km flyby, then no ΔV maneuver is required, that is, the cycler is ballistic. Otherwise, a ΔV maneuver is required, in which case we refer to the cycler as a powered cycler.

The Most Promising Solutions

Table 4 lists characteristics of the most promising cycler trajectories with $1 \leq n \leq 6$. Note that the Aldrin cycler (1L1) is among the most practical, despite the fact that the Earth flybys can not provide all of the required turning. Cycler 2L3 has a low V_∞ at Earth and Mars, but its aphelion is slightly below the orbit of Mars. It is the case 1 two-synodic-period cycler analyzed by Byrnes et al.[34]

Some of the cyclers with $n = 6$ are also promising. The 6S7, 6S8, and 6S9 cyclers have required turn angles that are less than the maximum possible turn angles; hence, they are ballistic cyclers. The required flyby periapsis altitudes at Earth are 1402, 5408, and 13,836 km, for the 6S7, 6S8, and 6S9 cycler, respectively. These cyclers also have a low V_∞ at Earth and at Mars. Unfortunately, 12 vehicles are required to provide short Mars–Earth and Earth–Mars trips every synodic period. Also, the 6S9 cycler has an aphelion at 1.40 AU, which is significantly below the orbital radius of Mars (at 1.52 AU).

Table 5 lists some characteristics of the $n = 7$ cyclers. These cyclers are special because they repeat every $T = nS = 7\cdot(2\frac{1}{7}) =$ 15 years, that is, after an integer number of years. Therefore, the Earth is at the same point in inertial space at the beginning and the end of the repeat interval T, so the cycler line of apsides does not need to be turned. Therefore, all $n = 7$ cyclers are ballistic cyclers. The VISIT 1 and VISIT 2 cyclers are $n = 7$ cyclers.

Many of the $n = 7$ cyclers encounter Earth and Mars more often than once every 15 years (Table 5). For example, the VISIT 1 cycler encounters Earth every 5 years and Mars every 3.75 years. An implication is that fewer than 14 spacecraft are required to ensure frequent short Earth–Mars transfers.

Also, because of their simple geometry, the orbital characteristics of the $n = 7$ cyclers can be found analytically. Because each $n = 7$ cycler makes r revolutions during the 15-year repeat time, the orbit period is 15/r years. In fact, an estimate of the period $\mathcal{P}$ (in years), of any nPr cycler is:

$$\mathcal{P} \approx 15n/\{7r + [n \text{ (mod 7)}]\} \tag{5}$$

Because the orbit period of an $n = 7$ cycler is 15/r years, the semimajor axis a (in AU) is

$$a = (15/r)^{\frac{2}{3}} \tag{6}$$

The cycler orbit perihelion radius R_p is not uniquely determined by r, however. If $r < 15$, then the semimajor axis of the cycler orbit is larger than the semimajor axis of Earth's orbit, so that all $R_p \in (0, 1)$ AU are possible. If we choose a value for R_p, then we can calculate the cycler orbit eccentricity e using

$$e = 1 - (R_p/a) = 1 - [R_p/(15/r)^{\frac{2}{3}}] \tag{7}$$

Table 4 The most promising cyclers that repeat every one to six synodic periods

Cycler, nPr	Aphelion radius, AU	V_∞ at Earth, km/s	V_∞ at Mars, km/s	Shortest transfer time, days	Required turn angle, deg	Maximum possible turn angle, deg
1L1[a]	2.23	6.54	9.75	146	84	72
2L2	2.33	10.06	11.27	158	134	44
2L3[b]	1.51[c]	5.65	3.05[d]	280[e]	135	82
3L4	1.89	11.78	9.68	189	167	35
3L5	1.45[c]	7.61	2.97[d]	274[e]	167	62
3S5	1.52[c]	12.27	5.45[d]	134[e]	167	33
4S5	1.82	11.23	8.89	88	167	38
4S6	1.53	8.51	4.07	157	167	54
5S4	2.49	10.62	12.05	75	134	41
5S5	2.09	9.08	9.87	89	134	50
5S6	1.79	7.51	7.32	111	135	62
5S7	1.54	5.86	3.67	170	135	79
5S8	1.34[c]	4.11	0.71[d]	167[e]	136	103
6S4	2.81	7.93	12.05	87	83	59
6S5	2.37	6.94	10.44	97	84	68
6S6	2.04	5.96	8.69	111	84	78
6S7	1.78	4.99	6.66	133	85[f]	90[f]
6S8	1.57	4.02	3.90	179	85[f]	104[f]
6S9	1.40[c]	3.04	1.21[d]	203[e]	86[f]	120[f]

[a]Aldrin cycler (see Ref. 14).
[b]Case 1 cycler analyzed by Byrnes et al.[34]
[c]Note: the semimajor axis of Mars is 1.52 AU.
[d]Difference between Mars's speed and spacecraft aphelion speed.
[e]Time to transfer from Earth to aphelion.
[f]Ballistic cycler: required turn angle is less than maximum possible turn angle.

Table 5 Cyclers that repeat every seven synodic periods (15 years)

Number of revolutions every 15 years, r	Period $(15/r)$, years	Aphelion radius,[a] AU	Years between Earth encounters	Years between Mars encounters
1	15	(11.16, 12.16)	15	15
2	7.5	(6.66, 7.66)	15	7.5
3	5	(4.85, 5.85)	5	15
4	3.75	(3.83, 4.83)	15	3.75
5	3	(3.16, 4.16)	3	15
6	2.5	(2.68, 3.68)	5	7.5
7	2.143	(2.32, 3.32)	15	15
8	1.875	(2.04, 3.04)	15	1.875
9	1.667	(1.81, 2.81)	5	15
10[b]	1.5	(1.62, 2.62)	3	7.5
11	1.364	(1.46, 2.46)	15	15
12[c]	1.25	(1.32, 2.32)	5	3.75
13	1.154	(1.20, 2.20)	15	15
14	1.071	(1.09, 2.09)	15	7.5

[a]Range given corresponds to perihelion range $R_p \in (0, 1)$ AU. [b]VISIT 2 cycler.[18] [c]VISIT 1 cycler.[18]

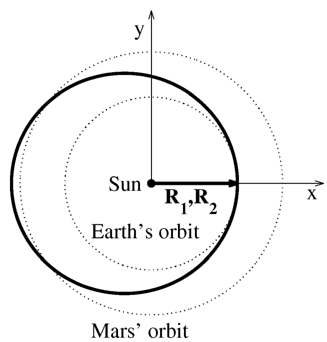


Fig. 4 VISIT 2 cycler 7(1.0)10.

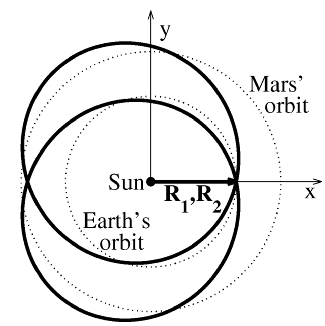


Fig. 5 VISIT 2 cyclers 7(0.9)10+ and 7(0.9)10–.

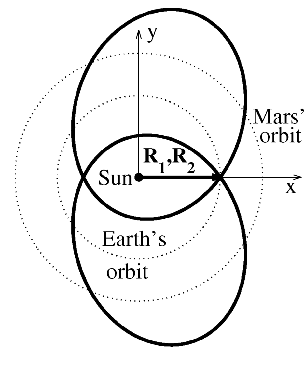


Fig. 6 VISIT 2 cyclers 7(0.5)10+ and 7(0.5)10–.

The cycler aphelion radius R_a can then be calculated using

$$R_a = a(1+e) = 2 \cdot (15/r)^{\frac{2}{3}} - R_p \tag{8}$$

When n is a multiple of seven, r and R_p determine the semimajor axis and eccentricity of the cycler orbit. However, there is still a degree of freedom in the argument of periapsis, ω, that is, the angle from the x direction to the cycler orbit periapsis direction. To determine the possible values of ω, we recall that the spacecraft orbit encounters the Earth when the Earth crosses the positive x axis. At that point, the distance from the sun to the spacecraft, R, is 1 AU, and the true anomaly of the spacecraft is $\pm\omega$, so that the conic equation for the spacecraft orbit tells us that

$$R(\pm\omega) = p/[1 + e\cos(\pm\omega)] = 1 \tag{9}$$

where $p = a(1-e^2)$ is the parameter of the cycler orbit. Hence, the argument of periapsis can have two possible values given by

$$\omega = \pm\arccos[(p-1)/e] \tag{10}$$

Therefore, when n is a multiple of seven, we denote a cycler using an expression of the form $n(R_p)r\pm$, where R_p is the perihelion radius (in AU) and the $+$ or $-$ indicates whether the argument of periapsis is positive or negative, respectively. Figures 4–6 illustrate the use of this notation for various VISIT 2 cyclers. In Fig. 4, the argument of periapsis is zero, and so the sign is not needed. We note that as the perihelion radius R_p is decreased, the V_∞ at Earth and Mars increases, so that a perihelion radius near 1 AU would tend to be chosen for practical applications.

Extending our Method of Constructing Cyclers

Although our method can construct many known cyclers, there still remain other known cyclers that our method cannot construct. Examples include the Rall[11] and Rall–Hollister[12] cyclers and various cyclers identified by Byrnes et al.[34] We anticipate that extensions of our method will be able to construct these cyclers as well.

Our method of constructing cyclers can be extended in several ways. For example, we have been assuming that all Earth encounters are an integer number of synodic periods apart, with no intermediate Earth flybys. There is no reason to rule out such intermediate Earth flybys. As long as the trajectory returns to Earth after an integer number of synodic periods, it is still a cycler. Indeed, intermediate Earth flybys could be very useful. If one gravity assist can not adequately turn the line of apsides, then more gravity assists might. Also, more Earth encounters may imply more short Earth–Mars transfers (or more short Mars–Earth transfers). (In the next section, we provide an example of a cycler using one intermediate Earth flyby.)

Other possible extensions to our method include 1) using gravity-assist maneuvers at Mars, Venus, or the moon; 2) allowing for inclined cycler orbits; and 3) allowing for propulsive ΔV maneuvers. A way of estimating the total ΔV required by powered cyclers would also be useful for identifying nearly ballistic cyclers.

Numerical Results for Cyclers in a More Accurate Model

To simplify the analysis, our method of constructing cyclers uses circular coplanar orbits for Earth and Mars. The hope is that any cycler found in the circular coplanar model would correspond to a similar cycler in a more accurate solar system model, that is, using analytic or integrated ephemerides for Earth and Mars.

As discussed earlier, the most promising cycler in Table 4 is the 1L1 cycler, which is better-known as the Aldrin cycler. We see in Table 4 that the required turn angle for 1L1 is 84 deg, but the Earth can only provide a turn angle of 72 deg, and so a ΔV maneuver is required. (Equivalently, the required flyby altitude is -1731 km, which must be raised to at least 200 km using a ΔV maneuver.) Previously, Byrnes et al.[14] minimized the required ΔV for both an inbound and an outbound version of the Aldrin cycler in a more accurate solar system model. Both trajectories were optimized over a 15-year time period (which is approximately how long it takes the Earth–Mars system to repeat inertially). They found that a ΔV maneuver was not required on every leg. The total ΔV required by the outbound cycler is 1.73 km/s (or 247 m/s per synodic period, on average), and the total ΔV required by the inbound cycler is 2.04 km/s (or 291 m/s per synodic period, on average). The flight times between Earth and Mars vary between 147 and 170 days. The V_∞ at Earth and Mars vary from 5.39 to 6.19 km/s, and from 6.05 to 11.74 km/s, respectively.

Cyclers 6S7 and 6S8 are also interesting because they are ballistic in the circular coplanar model and have sufficiently high aphelion radii. Our attempts to find long-term ballistic versions of these cyclers in a more accurate solar system model were unsuccessful.

The remaining cyclers in Table 4 are not practical. They either require too many vehicles, have overly long Earth–Mars transfer times, or have unacceptably high encounter V_∞. Moreover, none of them is ballistic, and some have an aphelion below the orbital radius of Mars. Therefore, we did not search for corresponding cyclers in a more accurate solar system model.

Some of the (seven-synodic-period) cyclers in Table 5 are worth investigating further because they are all ballistic and some have fairly frequent Earth and Mars encounters, particularly the VISIT 1 and VISIT 2 cyclers. In Ref. 35, Niehoff et al. provide a 20-year numerical solution to the VISIT 1 cycler that is ballistic in a realistic, that is, noncircular and noncoplanar, model of the solar system.

We have begun investigating extensions to our method to find other classes of cyclers. In one such extension, we allow for a single intermediate Earth flyby between two Earth encounters that are two synodic periods apart. Within that class of cyclers, we have found[36] a noteworthy new cycler that we call the ballistic S1L1 cycler. A 30-year itinerary for an outbound ballistic S1L1 cycler is given in Table 6. The positions and velocities of Earth and Mars were determined using the Jet Propulsion Laboratory's DE405 ephemerides. We note that the flyby V_∞ are all less than 7.7 km/s, the flyby altitudes are all greater than 7500 km, and the Earth–Mars legs range from 115 to 223 days.

Table 6 Outbound ballistic S1L1 cycler itinerary (using DE405 ephemerides of Earth and Mars)

Encounter	Date	Approach V_∞, km/s	Altitude of closest approach, km	Leg duration, days
Earth 1	9 June 2008	6.89 (launch)	——	541
Earth 2	3 Dec. 2009	6.90	31,114	186
Mars 3	6 June 2010	4.31	17,704	809
Earth 4	24 Aug. 2012	6.42	26,490	540
Earth 5	14 Feb. 2014	6.43	41,524	139
Mars 6	3 July 2014	7.14	12,179	890
Earth 7	9 Dec. 2016	4.01	27,726	530
Earth 8	22 May 2018	4.03	19,923	115
Mars 9	15 Sept. 2018	6.47	11,570	934
Earth 10	6 April 2021	4.61	22,992	532
Earth 11	20 Sept. 2022	4.59	14,780	223
Mars 12	1 May 2023	2.77	7,593	793
Earth 13	2 July 2025	7.08	23,858	542
Earth 14	26 Dec. 2026	7.09	35,164	170
Mars 15	14 June 2027	5.26	13,751	830
Earth 16	21 Sept. 2029	5.78	26,818	537
Earth 17	12 March 2031	5.78	39,044	125
Mars 18	15 July 2031	7.70	10,566	915
Earth 19	15 Jan. 2034	3.78	22,988	529
Earth 20	28 June 2035	3.76	9,586	138
Mars 21	13 Nov. 2035	4.68	15,525	907
Earth 22	7 May 2038	5.55	——	——

Conclusions

Earth–Mars cycler trajectories (cyclers) have the potential to be used in a future Earth–Mars transportation system. Until now, cyclers were found using a combination of intuition and numerical searching. We have developed a new, systematic method of constructing cyclers. It is based on the observation that if a spacecraft is on a cycler trajectory, then it must return to the Earth after an integer number of synodic periods. Our cycler construction method reveals that many previously known cyclers, such as the Aldrin cycler and the VISIT cyclers, are special cases of a more general family. Our method also finds new cyclers. However, none of the new cyclers appears to be as practical as the Aldrin cycler or the VISIT cyclers. The key contribution of this paper is a methodical technique to design cyclers.

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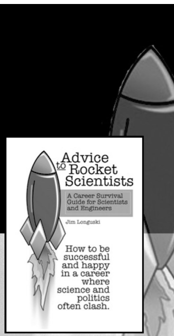

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